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CONSUMPTION, STOCK RETURNS,  
AND THE GAINS FROM  
INTERNATIONAL RISK-SHARING

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**ABSTRACT**

Standard theoretical models predict that domestic residents should diversify their portfolios into foreign assets much more than observed in practice. Whether this lack of diversification is important depends upon the potential gains from risk-sharing. General equilibrium models and consumption data tend to find that the costs are small, typically less than  $\frac{1}{2}\%$  of permanent consumption. On the other hand, stock returns imply gains that are several hundred times larger. In this paper, I examine the reasons for these differences. I find that the primary differences are due to either: (a) the much higher variability of stocks, and/or (b) the higher degree of risk aversion required to reconcile an international equity premium. On the other hand, the significant differences do not arise treating stock returns as exogenous.

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Domestic investors do not appear to hold a sufficient proportion of their wealth in foreign assets to diversify away domestic idiosyncratic risk. This is the conclusion of research using both consumption data and stock return data. General equilibrium models based upon complete markets imply that consumption growth rates across countries will be highly correlated. In fact, they have a low correlation, even lower than the correlation of output growth as shown by Backus, Kehoe and Kydland (1992).<sup>1</sup> Using stock return data, portfolios of domestic stocks and bonds are dominated in utility by portfolios that include foreign stocks with both higher mean and lower variance. Yet, the proportion of foreign equities in wealth held by US residents is significantly less than 10%.<sup>2</sup> These results imply that the degree of risk sharing by domestic investors is less than optimal. Since understanding why risk-sharing is imperfect is a central issue in economics, it has been the subject of a great deal of recent research, particularly in the international context.<sup>3</sup>

Whether the imperfect risk-sharing is surprising depends upon the size of potential gains from risk-sharing. Recent calculations of risk sharing based upon international consumption data suggest that these gains are quite small. For example, Cole and Obstfeld (1991) find that for representative consumers calibrated to U.S. data with time-additive constant relative risk aversion utility functions, the gains are less than ½% of permanent consumption for plausible parameter values.<sup>4</sup>

On the other hand, calculations of the gains from risk-sharing based upon stock returns give much larger estimates. The approach typically constructs combinations of domestic and foreign portfolios that minimize variance and maximize returns and asks whether domestic portfolios are dominated by these portfolios. In papers at least as early as Levy and Sarnat (1970), portfolios with

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<sup>1</sup>The relationship between complete markets and consumption correlations was first pointed out by Schienkman (1984) and Leme (1984).

<sup>2</sup>See French and Poterba (1991).

<sup>3</sup>For some papers in this area in international macroeconomics, see Obstfeld (1994b), Stockman and Tesar (1995), Lewis (1996), Baxter and Jermann (1993), and Baxter, Jermann, and King (1995).

<sup>4</sup>They also examine the effects of goods market prices on risk-sharing. For a recent survey of the welfare cost literature, see Tesar (1994).

foreign stocks were shown to strictly dominate domestic U.S. portfolios. Using utility functions similar to those used in the general equilibrium literature, I show below that this simple partial equilibrium framework implies welfare gains of at least 20% of permanent consumption.

In this paper, I address the question: why are the magnitudes of the gains based upon these two approaches so different? Several possibilities seem plausible and I consider each in turn.

First, consumption-based models typically assume common consumption growth rates across countries, while the stock-based models have focused upon differences in means as well as variances of stock returns. Thus, consumption-based models treat the gains of diversification as the minimization of risk associated with deviations around a common trend.<sup>5</sup> As Lucas (1987) has shown in a closed economy welfare calculation, the gains from minimizing consumption variations around a trend are small compared to the gains from increasing the consumption growth rate. To ask whether the assumption of common trend growth is important, I calculate the gains from the consumption model when growth rates are allowed to differ. However, I find that the gains for the US continue to be small.

Second, the differences may result from the general equilibrium relative to the partial equilibrium focus of the two literatures. The consumption-based literature has focused upon general equilibrium relationships that implicitly endogenize the effects of diversification on stock returns. The stock portfolio-based literature considers the marginal investor facing exogenous stock returns. Thus, it may be argued that the greater gains in diversification in stocks depend upon the investor not internalizing the effects of his actions on the equilibrium stock price.

Despite the plausibility of this explanation, I show that this effect does not always imply that partial equilibrium effects will be larger. Essentially, the stock return models allow the investor to rotate his consumption profile from its current level to a higher growth path. On the other hand, the general equilibrium models imply shifts in the current consumption level as well. As a result, the general equilibrium models can allow for intertemporal substitution. For high growth countries

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<sup>5</sup>An important difference is Obstfeld (1994b). This paper allows for different growth rates across countries with the presence of country-specific riskless interest rates. Calibrating the model with consumption data, Obstfeld finds that the welfare gains are large allowing for high risk aversion, a result I also find below.

such as Japan, the general equilibrium model allows these countries to trade off future consumption growth for a higher current consumption level. Therefore, this effect when stock prices are endogenized can actually provide an additional source of welfare gains.

Third, the different gains may simply arise from the stochastic processes used as the underlying measure of utility. In the stock market literature, the means and variances of stock returns are used as determinants of utility, whereas the consumption literature uses consumption means and variances. To examine the effects of these two different processes, I calibrate the consumption model with stock return statistics. Here I find that the certainty equivalent consumption growth paths across countries are sufficiently different that the gains from diversification can be very large. These divergent growth paths are principally driven by the extremely high variances of stock returns. Therefore, part of the answer to why the gains are so different appears to be that the risk-adjusted growth rates of consumption are much less divergent than those of stocks.

Fourth, it is well-known that the consumption-based general equilibrium models cannot duplicate the behavior of stock returns with plausible preference parameters. Therefore, the difference in gains may be a reflection of some outstanding asset pricing puzzles. Mehra and Prescott (1985) argue that the coefficient of relative risk-aversion must be implausibly high to explain the premium of equity over the risk-free rate in the U.S. Weil (1989) notes that, with time-additive utility, a coefficient of risk aversion large enough to match the equity premium implies an implausibly high risk-free rate. Thus, a finding of relatively low welfare costs associated with general equilibrium models calibrated to consumption may simply reflect the fact that these models cannot explain the expected equity returns observed.

I examine the possibility that differences in implicit preference parameters can explain the differences in welfare gains. I match the risk-aversion coefficient to moments of stocks and the risk-free rate. I find that when risk-aversion is sufficiently large to explain the equity premium on the optimal international stock portfolio, the gains from diversification measured with consumption are quite large. The reason is that high risk aversion dramatically increases the effects of the variance on certainty equivalent consumption. Thus, slight differences in consumption variances translate

into large differences in risk-adjusted consumption profiles. As a result, the gains from reducing the variance through diversification become very large.

Overall, the results show that the differences in welfare gains can largely be explained by differences in measured risk-adjusted consumption profiles across countries. These can come from large differences in actual variances in the underlying utility measure as in stock returns or from large differences in the utility value of variances such as when risk-aversion is very large.

In focusing upon why risk-sharing gains are so different, I necessarily ignore important issues and problems inherent in the existing literature. Thus, while the general equilibrium framework will summarize aspects of risk-sharing and intertemporal gains from trade existing in the literature, it will also contain well-known problems with explaining asset prices. Also, the stock return-based analysis will inherently treat historical means and variances as providing measures of the *ex ante* means and variances of investors, even though these contain significant variability.

In this paper, I use consumption and stock return data from the Group of Seven (G-7) countries to calculate the welfare gains. As such, I provide an examination of multilateral gains to risk-sharing. I then use Epstein-Zin-Weil preferences to allow the risk-aversion parameter to differ from the elasticity of intertemporal substitution in consumption. As pointed out by Cole and Obstfeld (1991) and van Wincoop (1991), and recently emphasized by Obstfeld (1994a,b), these two parameters play very different roles in welfare cost analysis.

The structure of the paper is as follows. In section 1, I describe the welfare gain function. In section 2, I use stock returns for the G-7 countries to examine stock returns using standard mean-variance analysis. In section 3, I develop a simple general equilibrium framework for examining risk-sharing based upon consumption. I then incorporate stock return data to calculate general equilibrium gains in section 4. In section 5, I use the expected stock returns to back out the implied utility parameters. With these values, I re-examine the welfare costs. Concluding remarks follow.

## **Section 1: The Gain Function**

### *(1.1) The Basic Framework*

I use a general gain function to analyze the differences in welfare gains between stock

return- and consumption- based models. Below, I assume that both consumption growth rates and stock returns are normally distributed so that their distributions are fully characterized by their means and variances.<sup>6</sup> I write this gain function generally as:

$$(1) \delta = \delta(\mu, \Sigma; \Omega; I)$$

where  $\mu$  is a vector of means and  $\Sigma$  is a variance-covariance matrix,  $\Omega$  is a set of preference parameters, and  $I$  is a function that indicates if the model is partial equilibrium or general equilibrium.

For the stock return-based model,  $\mu$  is the vector of stock-returns defined as  $\mu^s$ , while  $\Sigma$  is the variance-covariance of stock returns defined as  $\Sigma^s$ . On the other hand, for consumption-based calculations,  $\mu$  and  $\Sigma$  are the means and covariances of consumption growth rates, defined as  $\mu^c$  and  $\Sigma^c$ , respectively.

The set of preferences I consider include risk aversion,  $\gamma$ , the inverse of the intertemporal elasticity of substitution in consumption,  $\theta$ , and time preference,  $\beta$ . In the literature, plausible values for risk-aversion are considered to be between 1 and 10.<sup>7</sup> On the other hand,  $\theta$  is typically assumed to be rather high.<sup>8</sup> Finally,  $\beta$  is usually assumed to be less than one.<sup>9</sup> Below, I follow Cole and Obstfeld and treat  $\beta$  as equal to .98. I denote the set of parameters in this plausible range as  $\Omega^{\text{PLAUS}}$ .

Since these parameters cannot explain asset pricing relationships such as the high mean on equity relative to the risk-free rate, I also examine an alternative set of parameters. These parameters match certain moments on equities and the risk-free rate. I denote this set of

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<sup>6</sup>Some of the gains were also calculated based upon serially correlated means, giving similar results to those found below. Since the gains can only be simulated with serially correlated distributions, I assume i.i.d. returns in the text for tractability.

<sup>7</sup>See the discussion in Mehra and Prescott (1985). Risk aversion coefficients within this range are examined in studies of the welfare gains of international risk-sharing such as Obstfeld (1994a), Cole and Obstfeld (1991), and Tesar (1994). However, Kandel and Stambaugh (1991) argue that risk aversion is much higher, near 30. As I show below, even risk aversion as high as 30 is not high enough to explain the equity premium on international stocks.

<sup>8</sup>For example, Hall (1988) argues that  $\theta$  is probably not less than 10.

<sup>9</sup>See Kocherlakota (1990), however, for an argument that  $\beta$  can exceed one.

parameters as  $\Omega^{\text{MATCH}}$ .

Finally, the gain function depends upon whether the model is partial equilibrium or general equilibrium, endogenizing the stock price as a function of consumption. As I show below, this difference allows for intertemporal substitution in the form of a shift in the initial consumption level.

### *(1.2) Outline of the Analysis*

I begin by deriving the gain function and calculating its value for two benchmark cases. First, the stock-based partial equilibrium model implies the gain value:

$$(2) \delta = \delta(\mu^s, \Sigma^s; \Omega^{\text{PLAUS}}; \text{Partial}).$$

Thus, the gains depend upon the means and variances of stock returns. On the other hand, the general equilibrium model depends upon the means and variances of consumption growth rates:

$$(3) \delta = \delta(\mu^c, \Sigma^c; \Omega^{\text{PLAUS}}; \text{General}).$$

Typically, the consumption growth rates are assumed to be common across countries. As described in the introduction, the gains in (2) are much larger than those in (3).

I then investigate the reasons for these differences. I focus upon the general equilibrium gain and ask what assumption can make the gains match those of the partial equilibrium function. I first relax the assumption that mean consumption growth rates are common across countries. I next use stock-return moments (counterfactually) to calculate the general equilibrium gains:

$$(4) \delta = \delta(\mu^s, \Sigma^s; \Omega^{\text{PLAUS}}; \text{General}).$$

Finally, I study the effects of preferences parameters. I use the parameters that match asset return moments:

$$(5) \delta = \delta(\mu^c, \Sigma^c; \Omega^{\text{MATCH}}; \text{General}).$$

These experiments highlight the role played by aversion to high variances of the consumption profile, whether in the form of actual high variances in stocks as in (4) or in risk aversion as in (5).

### *(1.3) Calculating the Welfare Gains*

To calculate welfare gains, I follow Cole and Obstfeld (1991) in deriving welfare costs as a percentage of permanent consumption. They calculate the percentage of permanent consumption that must be taken away from an investor holding the optimal portfolio to make him indifferent to



holding the observed portfolio. Thus, if  $C_t$  is permanent observed consumption at time  $t$  and  $C_t^*$  is permanent consumption for the optimally diversified investor at time  $t$ , I calculate  $\delta$  defined by the relationship:

$$(6) \quad U_0(C_0) = U_0(C_0^*(1-\delta)).$$

Under the consumption-based approach, permanent consumption is calculated directly from consumption data. Under the stock-based approach, utility is defined as a function of wealth which is indirectly a function of stock returns.

#### (1.4) *The Utility Function*

Calculating welfare costs requires specifying a utility function. A common utility function used in the literature is time-additive constant relative risk aversion. This utility function restricts the coefficient of relative risk aversion to equal the inverse of the elasticity of intertemporal substitution in consumption. However, Obstfeld (1994a) shows that these two parameters play opposite roles in welfare calculations when the growth rate of risk-adjusted consumption is positive. Intuitively, higher risk aversion increases the value of diversification thereby raising welfare gains, while a lower elasticity of intertemporal substitution in consumption reduces the present value of gains in future consumption growth due to lower variance or higher mean.<sup>10</sup> Indeed, I find these two conflicting patterns in the results below.

To avoid restricting the parameters of risk aversion and intertemporal substitutability, I adopt the Epstein-Zin-Weil utility function that takes the recursive form:<sup>11</sup>

$$(7) \quad U_t = \{c_t^{1-\theta} + \beta [E_t(U_{t+1}^{1-\gamma})]^{(1-\theta)/(1-\gamma)}\}^{(1-\theta)}, \text{ for } \gamma, \theta > 0, \neq 1$$

When consumption is deterministic, the parameter  $\theta$  is the inverse of the intertemporal elasticity of substitution in consumption. On the other hand,  $\gamma$  is the parameter of relative risk aversion. The standard time-additive utility function results when  $\gamma = \theta$ . I use this utility function for both the stock-return based approach and the consumption-based approach below.

## **Section 2: Gains from International Risk-Sharing with Exogenous Stock Returns**

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<sup>10</sup>Technically, the effect of changes in intertemporal substitution depends upon the growth rate of risk-adjusted consumption being positive. See Obstfeld (1994a).

<sup>11</sup>For more discussion of this utility function, see Epstein and Zin (1989) and Weil (1990).

The gains from international diversification in stocks have been noted since at least the 1970s. A standard approach for examining these gains is to calculate the mean and variance of portfolios that include foreign stocks and determine whether they dominate portfolios of domestic stocks alone.<sup>12</sup> Typically, portfolios with foreign stocks absolutely dominate portfolios based upon domestic stocks alone. That is, portfolios with foreign stocks imply either lower variance or higher return or both.

### *(2.1) The Basic Framework*

To illustrate, Figure 1 depicts a combination of mean returns and standard deviations of portfolio combinations that allow for an increasing weight on foreign stocks in the portfolio of a domestic U.S. investor. In particular, I take the returns on the stock market indices from Morgan Stanley International Capital Market Perspectives for the G-7 from 1969 to 1993. I then construct a mutual fund by taking a population-weighted average of the non-US countries, converting the foreign returns into dollars and then deflating by the U.S. price level. Details are provided in the data appendix. Point A represents the mean and standard deviation of the U.S. stock market over the period, corresponding to a zero weight on foreign stocks. Moving along the curve represents higher weights to the foreign stock. Clearly, the U.S. stock market is dominated by including foreign stocks. Point B represents a portfolio of 5% holdings of foreign stocks, consistent with the findings of French and Poterba (1991).

The portfolios represented by Figure 1 therefore are combinations of the U.S. stock market and a fixed portfolio of foreign stocks. A fully optimal combination of foreign stocks would be the portfolios providing the lowest variance for any given mean return, the so-called "efficient frontier." Since the portfolios of this efficient frontier would imply even higher utility than those given by this risk-return tradeoff, the true gains from stock diversification will be even higher than those measured by Figure 1.

### *(2.2) Evaluating Utility*

With the utility function in (7), I calculate the gains from moving from the utility of a portfolio of 100% U.S. stocks at point A to the utility at the optimal combination. To calculate the

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<sup>12</sup>For a recent survey, see Lewis (1995).

optimal allocation, I must maximize utility in equation (1) with respect to returns. Following standard practice, I treat returns as i.i.d. I assume that domestic U.S. returns,  $R_t^u$ , and returns on the optimal portfolio,  $R_t^*$ , are log-normally distributed:  $\ln(R_t^u) \sim N(\mu_u - \frac{1}{2}\sigma_u^2, \sigma_u^2)$ ,  $\ln(R_t^*) \sim N(\mu_* - \frac{1}{2}\sigma_*^2, \sigma_*^2)$ .<sup>13</sup> Then since (7) is homothetic, the value function is proportional to returns and, since wealth is proportional to returns, the value function is also proportional to wealth. Therefore, for an investor holding only the U.S. stock market,

$$(8) \quad E_t U(C_t) = W_t (1 - \beta \exp[(1-\theta)(\mu_u - \frac{1}{2}\gamma \sigma_u^2)])^{-1/(1-\theta)}$$

where  $W_t$  is the investor's wealth at time  $t$ . Following standard mean-variance analysis, I treat the initial wealth level as exogenously given.

As described in Obstfeld (1994a), the inverse of  $\beta \exp[(1-\theta)(\mu_u - \frac{1}{2}\gamma \sigma_u^2)]$  operates as an overall discount rate for future returns.  $\beta$  is the parameter of time-preference in the non-stochastic time-additive case. This term is multiplied by the exponential of the risk-adjusted growth rate of  $\mu_u - \frac{1}{2}\gamma \sigma_u^2$ . The measure of relative risk aversion  $\gamma$  multiplies the variance since the returns follow a stochastic trend. In this case, shocks to returns persist into the future so that the variance cumulates over time. In turn, the risk-adjusted growth rate is multiplied by  $(1 - \theta)$ , where  $\theta$  is the inverse of the elasticity of intertemporal substitutability for non-random consumption paths. As  $\theta$  increases, so that substitutability decreases, the future becomes less important and the discount rate decreases. The impact of higher  $\theta$  on lowering the utility value of future consumption is important for understanding the effects of  $\theta$  on welfare gains below.

The expected utility function in (8) is well-defined only when the discount rate is less than one; i.e.,  $1 > \beta \exp[(1-\theta)(\mu_u - \frac{1}{2}\gamma \sigma_u^2)]$  or equivalently,

$$(9) \quad \beta^{-1} > \exp[(1-\theta)(\mu_u - \frac{1}{2}\gamma \sigma_u^2)].$$

Otherwise, the rate of growth of utility does not converge as  $t$  goes to infinity. For example, as  $\gamma$  become large, it is clear that the risk-adjusted growth rate becomes negative even for  $\mu_u > 0$ . This

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<sup>13</sup>If the optimal portfolio is a linear combination of log normal distributions, then this assumption will clearly not hold. However, since the purpose of this section is to illustrate typical gains, I will maintain this assumption for simplicity. Below, I consider an equilibrium of time-varying weighted averages of log-normal returns and examine its distributional characteristics with Monte Carlo experiments.

relationship can be re-enforced by high values of  $\theta$  and high variances, as I find in stocks below.

For investors holding optimally diversified stocks, the expected utility is similarly:

$$(10) U_i(C_i^*) = W_i (1 - \beta \exp[(1-\theta)(\mu_* - \frac{1}{2} \gamma \sigma_*^2)])^{-(1/(1-\theta))}.$$

Figure 1 shows the points that maximize equation (10) for different levels of  $\gamma$  and  $\theta$ . For the time-additive case where  $\gamma = 2$ , investors prefer to hold all of their stocks in the foreign equity at point Z. As  $\gamma$  increases, the optimal holdings of domestic stocks increase. When  $\gamma = 4$  depicted at point Y, domestic investors choose to hold only 78% of their portfolio in foreign stocks.

### (2.3) Calculating Welfare Gains

To calculate the welfare gains, I equate the utility of consumption under autarchy to the utility of  $(1-\delta)$  times consumption under the optimal portfolio, as described in equation (6). Therefore, I set equation (8) equal to  $(1-\delta)$  times equation (10) and solve for  $\delta$  at time 0. This gives:

$$(11) \delta = 1 - \{E_0[U(C_0^*)]/E_0[U(C_0)]\}^{(1/(1-\theta))}$$

$$= 1 - \frac{(1 - \beta \exp[(1-\theta)(\mu_* - \frac{1}{2} \gamma \sigma_*^2)])}{(1 - \beta \exp[(1-\theta)(\mu_{us} - \frac{1}{2} \gamma \sigma_{us}^2)])}.$$

This equation defines the partial equilibrium gain function,  $\delta(\mu, \Sigma; \Omega; \text{Partial})$ .

Table 1 reports the estimates of the welfare costs using stock return means and variances for different parameters of the utility function. Panel A reports welfare costs for the case of time-additive utility where  $\gamma = \theta$  for a "plausible" range of risk aversion. For low values of relative risk aversion between 2 to 4, the welfare gains relative to the diversified portfolio range from 18% to 29% of permanent consumption. As noted above, these measures represent the lower bounds for the true gains since the feasible set of portfolios is restricted to linear combinations of the U.S. stock market and a fixed mutual fund of foreign stocks. The gains from diversifying into foreign stocks therefore are quite high. For  $\gamma = \theta = 5$ , condition (9) is violated and expected utility is not defined.

As Panel A shows, the welfare gains do not rise or fall monotonically with risk aversion. A movement from  $\gamma = 2$  to  $\gamma = 3$  results in a fall in welfare gains, but a further rise in  $\gamma$  to 4 implies a rise in welfare gains. This behavior is an artifact of constraining the measure of relative

risk aversion,  $\gamma$ , to equal the inverse of the coefficient of intertemporal substitutability,  $\theta$ . As the expected utilities in (8) and (10) show, for given means and variances, expected utility is decreasing in risk-aversion,  $\gamma$ . Higher levels of risk-aversion make the investor more averse to uncertainty and, hence, lower utility. Therefore, international diversification implies greater welfare gains. On the other hand, higher values for  $\theta$ , the inverse of the measure of intertemporal substitutability, means that the marginal utility of consumption is falling more quickly over time. This effect diminishes the welfare benefit of a given (positive) risk-adjusted growth rate. Therefore, higher values of  $\theta$  imply lower welfare gains. The results in Panel A confound both effects so that welfare gains first fall and then rise for the cases where  $\gamma = \theta$ .

Panel B shows the effects of relaxing the restriction that  $\theta = \gamma$ . For given  $\theta$ , the gains are increasing in risk aversion. On the other hand, for given  $\gamma$ , the gains are declining as the substitutability in consumption declines; i.e., as  $\theta$  increases. Overall, the gains are very high, ranging from 10% to 52% of permanent consumption.

#### *(2.4) Graphical Description of the Gains from Diversification*

Figure 2 illustrates the gains to an individual US investor from moving from the consumption growth path associated with domestic returns with the path associated with a portfolio including foreign returns. The highest two lines show the difference in growth paths associated with holding the foreign relative to the domestic portfolio in the case where there is no uncertainty. Clearly, the higher returns, labeled  $\mu^*$ , correspond to the investor with the optimal portfolio having a much higher consumption profile than the lower mean US returns, labeled by  $\mu$ .

The lower two lines show the certainty equivalent paths in the presence of uncertainty. Both paths are lower than their counterparts in the absence of uncertainty. However, the portfolio with foreign returns also has lower variance than the domestic portfolio. As a result, the consumption profile falls by less.

Despite the simplicity of this experiment, it illustrates the frequently-heard claim that the marginal U.S. investor would benefit enormously from diversifying into foreign stocks. Clearly, this analysis treats the stock returns as exogenous. Therefore, the investor's behavior does not affect

the returns. This assumption contrasts markedly from the general equilibrium models used to measure welfare costs using consumption, as I consider below.

### Section 3: Gains from International Diversification with Endogenous Stock Returns

I now provide a simple framework for assessing welfare costs based upon general equilibrium models calibrated with consumption behavior. Following the literature, I assume that identical representative agents in country  $j$  for each of  $N$  countries receive their own country's *per capita* endowment stream,  $e_t^j$ . As before, I calculate welfare gains by solving for and comparing the certainty equivalent consumption paths in the absence of risk-sharing and under perfect risk-sharing. To do so, I first briefly review the closed economy case in the absence of international risk sharing and then construct the diversified equilibrium below.

#### (3.1) Autarchy

The equilibrium consumption process without access to international markets is trivially given by the endowment process. For countries with identical preferences, the pricing of risk will differ in the  $N$  closed economies if the endowment processes differ across countries. This closed economy equilibrium is well-known and therefore is only briefly summarized here.<sup>14</sup>

Defining  $s_t$  as the state of the economy at time  $t$ , including realizations of the endowment, the representative agent will maximize utility in (7) such that his budget constraint holds. Specifically, the consumer-investor will consume each period and buy shares in the domestic stock. Thus, the optimization is given by the Bellman equation:

$$(12) \quad V(w_t^j, s_t) = \text{Max}_{c_t^j, \kappa_t^j} [(c_t^j)^{(1-\theta)} + \beta E_t(V(w_{t+1}^j, s_{t+1}))^{1-\gamma}]^{(1-\theta)/(1-\gamma)}$$

s.t.

$$(13) \quad c_{t+1}^j + \kappa_{t+1}^j q_{t+1}^j = (q_{t+1}^j + e_{t+1}^j) \kappa_t^j$$

where,  $w_t^j \equiv \kappa_t^j q_t^j$  is the wealth of country  $j$  investor at time  $t$ ,  $s_t$  is the state vector at time  $t$ ,  $\kappa_t^j$  are his shares held of stocks paying out the *per capita* endowment of country  $j$  and  $q_t^j$  is this country's

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<sup>14</sup>See Lucas (1978, 1982) for the time-additive, expected utility case. For asset pricing with the preferences examine here, see Epstein (1988).

stock price. The first-order condition for this maximization with i.i.d. returns is:<sup>15</sup>

$$(14) \quad \beta^{(\gamma-1)/(\theta-1)} E_t\{(c_{t+1}^j/c_t^j)^{-\theta(\gamma-1)/(\theta-1)} (1 + R_{t+1}^j)^{(\gamma-1)/(\theta-1)}\} = 1.$$

Defining the return on the domestic stock paying out domestic *per capita* endowments,

$$(15) \quad R_t^j = (e_t^j + q_t^j)/q_{t-1}^j.$$

Following the analysis in section 1, I assume that the endowments are i.i.d. Clearly, this assumption is only an approximation. In Lewis (1994), I simulate serially correlated endowment processes with results similar to the findings below.

I also assume the endowments are log-normally distributed so that  $\ln(e_t^j/e_{t-1}^j) \sim N(\mu^j - \frac{1}{2}(\sigma^j)^2, (\sigma^j)^2)$ . Using this assumption, substituting (15) into (14) and solving for the stock price,  $q_t^j$ , implies:

$$(16) \quad q_t^j = e_t^j \{\beta M_j^{(1-\theta)}/(1 - \beta M_j^{(1-\theta)})\}$$

where  $M_j = \exp(\mu_j - \frac{1}{2} \gamma \sigma_j^2)$  is the exponential of the risk-adjusted growth rate. As described before,  $\mu_j - \frac{1}{2} \gamma \sigma_j^2$  is the certainty equivalent consumption growth path, this time at autarchy. In equilibrium,  $\kappa_t^j = 1$  so that each investor holds one share of the endowment of *per capita* output.

### (3.2) Risk-Sharing with Open Financial Markets

Suppose now that each country can trade in the equities of all other countries. Thus, in the new equilibrium, investors in country  $j$  hold  $\kappa^{ij}$  shares in stocks of country  $i$  endowments. Instead of (13), the budget constraint becomes:

$$(17) \quad c_t^j + \sum_{i=1}^N \kappa_t^{ij} q_t^i \leq \sum_{i=1}^N \kappa_{t-1}^{ij} (e_t^i + q_t^i)$$

where the maximization in (12) is now over  $c_t^j$  and  $\kappa^{ij}$ ,  $\forall i = 1, \dots, N$ .

Since all countries have the same homothetic utility functions, then each country holds the same portfolio allocation among equities in equilibrium.<sup>16</sup> Therefore, the problem can be written equivalently in terms of a world mutual fund paying out the world *per capita* endowment, defined as  $\underline{e}_t$ . Shares of the mutual fund held by country  $j$  are defined as  $\underline{\kappa}_t^j$  and its price is  $\underline{q}_t$ . The budget constraint can therefore be rewritten as:

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<sup>15</sup>For a discussion of this first-order condition and its relationship with the standard time-additive case, see Epstein (1988).

<sup>16</sup>For a proof, see, for example, Ingersoll (1987).

$$(17') \quad c_t^j + \kappa_t^j \leq \kappa_{t+1}^j (\underline{e}_t + \underline{q}_t).$$

This maximization implies the first order condition,

$$(18) \quad \beta^{(\gamma-1)/(\theta-1)} E_t\{(c_{t+1}^j/c_t^j)^{\theta(\gamma-1)/(\theta-1)} (1 + \underline{R}_{t+1})^{(\gamma-1)/(\theta-1)}(1 + R_{t+1})\} = 1,$$

where  $\underline{R}_t$  is the return on the optimal mutual fund portfolio and  $R_{t+1}$  is the return on any asset.

Investors in each country sell the stream of *per capita* endowments in their own country for the stream of the world *per capita* endowments. Thus,  $\kappa_t = \underline{q}_t^j/\underline{q}_t$  where  $\underline{q}_t^j$  is the stock price of country  $j$  sold on world markets. As above, the welfare gain from diversifying is defined as the percentage of permanent consumption that must be taken away from an individual at the optimum to make him indifferent between risk-sharing or not. Thus, if  $C_0^j$  is the permanent consumption at time 0 for country  $j$  and  $C_0^{j*}$  is its permanent consumption under the optimal risk-sharing policy, then the welfare gain  $\delta^j$  is given by the following relationship:

$$(19) \quad E_0 U\{C_0^j\} = E_0 U\{(1-\delta^j) C_0^{j*}\} \equiv E_0 U\{(1-\delta^j) (\underline{q}_0^j/\underline{q}_0) \underline{C}_0\}$$

where  $\underline{C}_0$  is the stream of world *per capita* endowments. The second equality follows since the optimal consumption path equals the amount of the world *per capita* endowment that can be bought by country  $j$ ,  $(\underline{q}_0^j/\underline{q}_0) \underline{C}_0$ .

Calculating  $\delta^j$  in (19) requires deriving the stock prices,  $\underline{q}^j$  and  $\underline{q}$ . As above, I assume that endowment processes are log-normally distributed. I further make the assumption that the world *per capita* endowment process can be approximated by a log-normal distribution so that:  $\ln(\underline{e}_t/\underline{e}_{t-1}) \sim N(\underline{\mu} - \frac{1}{2} \underline{\sigma}^2, \underline{\sigma}^2)$ . This assumption ignores the relationship between country *per capita* endowments and world *per capita* endowments:  $\underline{e}_t = \sum_{j=1}^N \pi_t^j e_t^j$ , where  $\pi_t^j$  is the share of world population in country  $j$  at time  $t$ . However, the appendix provides Monte Carlo simulations that show the true world endowment distribution implied by log-normal country endowments and the population shares in the data is insignificantly different from a log-normal distribution. With this apparently innocuous assumption, the problem has a simple and tractable analytical solution.

The stock prices are then straightforward to derive. The world mutual fund is analogous to the closed economy stock price in (16):

$$(20) \quad \underline{q}_t = \underline{e}_t \{\beta \underline{M}^{(1-\theta)} / (1 - \beta \underline{M}^{(1-\theta)})\}$$



where  $\underline{M} = \exp(\underline{\mu} - \frac{1}{2} \gamma \underline{\sigma}^2)$ , the exponential of the risk-adjusted growth rate of the world endowment.  $\underline{M}$  now captures the certainty equivalent consumption growth rate for all countries, since the intertemporal marginal rate of substitution is the same in the integrated market.

Country  $j$ 's stock price on world markets depends upon the expected value of the product of future marginal utility of the world endowment and the realization of country  $j$ 's endowment:

$$(21) \quad q^j = e_t^j \{ \beta \underline{M}^\theta H_j / (1 - \beta \underline{M}^\theta H_j) \}$$

where  $H_j$  captures this interaction. In particular,  $H_j = \exp\{\mu_j + \frac{1}{2} \gamma \underline{\sigma}^2 - \gamma \underline{\sigma}_j\}$  where  $\underline{\sigma}_j$  is the covariance between the country  $j$  endowment growth and the world endowment growth. This factor captures the degree to which the expected dividends payed out by the stock hedge against the realizations of marginal utility from the world mutual fund. Other things equal, a country's stock price will increase with its mean growth rate,  $\mu_j$ , and decrease with its covariance with the world endowment,  $\underline{\sigma}_j$ . As for the world mutual fund, the country's stock price in (21) depends upon the common risk-adjusted growth rate,  $\underline{M}$ , raised to the factor  $\theta$ .<sup>17</sup>

Given these stock prices for each country and the mutual fund price, I now determine the welfare gain. Substituting (20) and (21) into (19) and using the solution for the value function as in (8), the welfare gain is:

$$(22) \quad \delta^j = 1 - (e_0^j \underline{q}_0 / \underline{e}_0 \underline{q}_0^j) E_0\{U(\underline{C}_0)/U(C_0^j)\}^{(1/(1-\theta))} \\ = 1 - \frac{(e_0^j \underline{q}_0) (1 - \beta \exp[(1-\theta)(\underline{\mu} - \frac{1}{2} \gamma \underline{\sigma}^2)])}{(\underline{e}_0 \underline{q}_0^j) (1 - \beta \exp[(1-\theta)(\mu_j - \frac{1}{2} \gamma \sigma_j^2)])}$$

Thus, the welfare gains depend both upon the utility under autarchy relative to the optimal world portfolio, as well as the ratio of the value of domestic equity,  $\underline{q}^j$ , to world equity,  $\underline{q}$ .

This equation defines the general equilibrium gain function,  $\delta(\mu, \Sigma; \Omega; \text{General})$ . Comparing this function with the partial equilibrium gain function in (11) shows that they have an identical form when the ratio of endowments and stock prices equal one; i.e., when  $(e_0^j \underline{q}_0 / \underline{e}_0 \underline{q}_0^j) = 1$ . As I show below, this difference provides an avenue for intertemporal substitution in the case of the

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<sup>17</sup>It is easy to verify that (21) reduces to (20) for the case where the "country" is the world so that  $\mu_j = \underline{\mu}$  and  $\sigma_j = \underline{\sigma}$ .

general equilibrium framework.

### *(3.3) The Empirical Gains*

Table 2 reports the results of calculating the welfare gains for each of the G-7 countries against the world. Following Obstfeld (1994b), I use the data in the Penn World Tables (Summers and Heston (1991)), except updated through 1992. These data give consumption data in common units over time and countries. For the purpose of comparison with the stock market data available, the sample begins in 1969. Panel A reports summary statistics for the G-7 countries.

Many studies treat mean consumption growth rates across countries as equal. For this reason, Panel B reports the welfare gains from risk-sharing when all countries are assumed to share the same growth rate as the world. To conserve space, the gains are reported for the two extreme cases examined in Table 1, i.e., for  $\gamma$  and  $\theta$  equal to 2 and 5. The lowest welfare gains are represented by the case where risk aversion is lowest at  $\gamma = 2$  and where the inverse of intertemporal substitutability is highest at  $\theta = 5$ . The gains are slightly larger for the time-additive cases,  $\gamma = \theta = 2$  and 5, and are largest when risk aversion and intertemporal substitution is high with  $\gamma = 5$  and  $\theta = 2$ . However, the gains are significantly lower than those in Table 1 based upon similar utility parameters.

In most cases, the costs are quite low. For the U.S., the maximum gain is 0.25% of permanent consumption. Certain countries suggest higher gains from risk-sharing relative to the other countries. These countries either have higher variation in consumption relative to the rest of the world such as Canada and the U.K. or else have a lower consumption correlation relative to the rest of the world such as Italy.

Panel C examines the effects of allowing the mean growth rates to differ. Strikingly, the maximum welfare gains for Japan with differing means increase to 5.5% of permanent consumption in Panel C from 0.95% in Panel B. Thus, much of the gains to Japan appear to derive from the strong equity value of its high growth. This effect will be examined graphically below.

The overall magnitude of the gains tend to increase with differing means. For the lowest gain parameters, the U.S. gains increase to 0.06% from 0.04% of permanent consumption. These

gains are largest for Japan at about 2%, dramatically higher than the 0.15% gains when the means are the same. Although these values are larger than when the means are assumed to be the same, they remain significantly smaller than the stock return gains given in Table 1.

### (3.4) Graphical Representation of Gains

Figure 3 shows the source of these gains with differing means for the low gain case where  $\gamma = 2$  and  $\theta = 5$ . The other parameter values imply similar trade-offs. For each country, the figure shows the certainty equivalent consumption growth paths for autarchy as the dashed line (labeled as " $\mu - \frac{1}{2} \gamma \sigma^2$ ") and for risk-sharing as a solid line (labeled as " $\mu - \frac{1}{2} \gamma \sigma^2$ "). The intercepts show the level of consumption associated with the path at time 0.

This figure makes clear why Japan has relatively large gains while the US has low gains. The Japanese sell off their claims to their high growth economy by receiving a relatively high price in the beginning of time. They are happy to do this because they would like to intertemporally substitute future consumption for current consumption. The rest of the world wants the higher growth rate and are willing to provide this substitution. Therefore, Japan enjoys large gains.

On the other hand, the US has low gains because Americans trade off current consumption for future consumption. Furthermore, since the US endowment stream is highly correlated with the world endowment process, the price of US equity is relatively low. Similar trade-offs are shown for the other countries as well.

### (3.5) Conclusions from Consumption Gains

Overall, the evidence in Table 2 and Figure 3 shows that the welfare gains from international risk-sharing using consumption data in a general equilibrium framework are significantly lower than those found in Table 1 using stock market returns in a partial equilibrium framework. However, the gain function in (22) calculated from the consumption means and variances is similar in form to the gain function in (11) calculated from the stock return means and variances. There are two differences.

First, the consumption-based gain incorporates the value of the stock price on world gain parameters, the U.S. gains increase to 0.06% from 0.04% of permanent consumption. These

missing in the stock-based approach. But the intertemporal substitution can either mitigate or exacerbate the gains depending upon whether the country is a low or high risk-adjusted growth country. Therefore, this explanation alone does not appear to explain the puzzle.

Second, the measures of the risk-adjusted growth paths are different. The consumption-based approach calculates this path from the means and variances of consumption growth. The stock-based gain generates a certainty equivalent consumption profile using the means and variances of stock returns. This approach treats the underlying consumption process as being generated by the stock return process. This distinction suggests a simple, counterfactual experiment for studying the implications of this assumption, described below.

#### **Section 4: Using the Stock Return Distribution to Calculate Consumption-Based Gains**

To investigate the effects of measuring the consumption process with stock returns, I re-examine the general equilibrium gain function. Instead of the means and variances of consumption growth in the gain function (22), I use the means and variances of stock return growth. Although calculating the gains in this way is counterfactual, it addresses the question: are the higher gains measured by stocks generated by the higher variances and/or greater variation in means than those of consumption growth?

##### *(4.1) Measuring the Gains*

In this section, I calculate the general equilibrium gains using the means and variances of stock returns,  $\delta(\mu^*, \Sigma^*, \Omega^{\text{PLAUS}}, \text{General})$ . Panel A of Table 3 reports the summary statistics used in the calculation. As with consumption, the correlation of US equity returns with the world portfolio is higher than that of the other countries.

Panel B gives the gains for the extreme case of low and high gain parameter combinations:  $\theta = 5, \gamma = 2$  and  $\theta = 2, \gamma = 5$ . Clearly, these gains are substantially larger than those in Table 2. As with using consumption means and variances, the gains are lower for the US than the other countries since its correlation with the world is higher. For the other countries, the gains are quite large. Even for the low gain parameter case, the benefits to these countries from risk-sharing range from about 15% to 26% of permanent consumption.

#### *(4.2) Graphical Representation of the Gains*

Figure 4 shows why these gains are so much larger when stock return means and variances are used. As before, the figure shows the autarchy certainty equivalent growth path of consumption as the dashed line, while the risk-sharing path is depicted by the solid line. As in Figure 3, the parameter values are the low gains case:  $\gamma = 2$  and  $\theta = 5$ .

For countries such as Canada, the differences in variances based upon risk-sharing relative to not risk-sharing imply significant increases in the tilt to risk-adjusted consumption growth. Since Italy's autarchy growth rate is negative, the gains from moving to risk-sharing are unbounded and undefined. For France, Germany, and the United Kingdom, risk-sharing not only puts the economies on a higher risk-adjusted consumption growth path, it also raises initial consumption as well. The reason is that these countries obtain a high price for their equity on world markets because of their relatively high growth rate and low correlation with the world endowment. Even though their growth rates are higher than the world endowment, they achieve a higher risk-adjusted growth rate of consumption because of the lower variance resulting from diversification. This improvement in variance measured with consumption data is significantly lower since consumption variances are so much smaller.

As in Figure 3, Japan gains by intertemporally substituting higher current consumption levels for future consumption growth. Using stock return means and variances exacerbates this effect, however. The significantly higher mean of Japanese stocks together with its diversification benefits imply that Japan can command a significantly high price of stocks on world markets. With a growth rate of nearly 11% in the stock index, the Japanese stock price is so high that the risk-sharing consumption levels are above the autarchy levels for over 25 years. As a result, the gains are substantial.

As this figure shows, an important contributing factor to why stock returns suggest such different gains of international risk-sharing is the significantly higher variance of stocks. This high variance implies both that investors are willing to pay significantly for claims on equities that reduce overall consumption risk and that certainty equivalent consumption is dramatically increased by

reductions in variance.

These calculations are counterfactual, however. They treat stock returns as though they characterize the consumption process. It is well-known that the general equilibrium framework cannot explain the means or variances of stock returns with plausible utility parameters. Therefore, an alternative explanation of the puzzle may be that the preference parameter values for the consumption-based model are inconsistent with the behavior of stock returns. I consider this possibility next.

### **Section 5: Using Preference Parameters Implied by Matching Asset Returns**

The inconsistency between stock return behavior and its predictions from consumption-based models using conventional utility parameter values has been established repeatedly in the literature. For example, Mehra and Prescott (1985) show that, with time-additive utility, the mean excess return on equity over the risk-free rate in the U.S. requires very high levels of risk-aversion. This relationship is verified in Weil (1989) who shows that high levels of risk-aversion also generate implausibly high levels of the risk-free rate. Kandel and Stambaugh (1991) argue that, while the equity premium and the risk-free rates can be explained by high levels of risk-aversion, other features of stock returns cannot.

In this section, I ask whether parameter values that match features of stock return behavior can help reconcile the difference in implied gains between stock returns and consumption risk-sharing.

#### *(5.1) Matching the Equity Returns and Risk-Free Rate*

I begin by matching the risk-aversion parameters to the returns on stocks and a risk-free rate. Setting  $R_t$  in Euler equation (18) to the risk-free rate,  $R_t^r$ , implies:

$$(23) \quad R_t^r = \beta^{-1} \exp(\theta \underline{\mu} - (1 + \theta) \frac{1}{2} \gamma \underline{\sigma}^2).$$

As long as the risk-adjusted growth rate is positive, the risk-free rate is increasing in  $\theta$  (decreasing in intertemporal substitutability). On the other hand, as risk-aversion increases, the risk-free rate decreases due to the precautionary savings effect.

I calculate implied stock returns by substituting the stock price solutions (21) and (22) into

the definition of returns (15), and the resulting expression into the Euler equation (18).<sup>18</sup> The unconditional means of these returns are:

$$(24) \ E(\underline{R}_t^j) = \beta^{-1} \exp(\theta \underline{\mu} - (\theta+1) \frac{1}{2} \gamma \underline{\sigma}^2 + \gamma \underline{g}_j)$$

for country returns, and

$$(25) \ E(\underline{R}_w) = \beta^{-1} \exp(\theta \underline{\mu} - (\theta-1) \frac{1}{2} \gamma \underline{\sigma}^2)$$

for the world mutual fund. It will be useful to compare the equity premium of the world mutual fund to the closed economy equity premium for the individual countries in autarchy. The equity return for the closed economy is:

$$(26) \ E(R_t^j) = \beta^{-1} \exp(\theta \mu_j - (\theta-1) \frac{1}{2} \gamma \sigma_j^2).$$

Table 4 reports the combinations of the risk-aversion parameter,  $\gamma$ , and the intertemporal substitutability parameter,  $\theta$ , that matches the equity return and risk-free rate observed in the data to the mean implied returns. For the purposes of calculating the risk-free rate, I use the dollar London-Interbank Offer Rate (LIBOR) deflated by U.S. inflation. I treat the risk-free rate as common across countries to examine an equilibrium where international investors can borrow at the same rate but may not necessarily acquire foreign equity.<sup>19</sup>

I then match the parameters in the utility function to the autarchy equilibrium by setting the means of empirical equity returns in each country and the risk-free rate to equal the means in the model. For this and all other experiments, I assume a time preference parameter of .98, following Obstfeld (1994a.) Panel A gives the result when the autarchy model returns are matched to the empirical U.S. equity returns and risk-free rate.<sup>20</sup> As the panel shows, the returns are more uniform across countries than in the actual data given in Panel A of Table 3. Mean Japanese equity

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<sup>18</sup>Returns on stocks in the integrated world portfolio are:  $\underline{R}_t^j = (e_t^j/e_{t-1}^j) \beta^{-1} \underline{M}^{(\theta-\gamma)} H_j^{-1} = (e_t^j/e_{t-1}^j) \beta^{-1} \exp(\theta \underline{\mu} - (1+\theta) \frac{1}{2} \gamma \underline{\sigma}^2 - \mu_j + \gamma \underline{g}_j)$  for individual stocks and  $\underline{R}_t = (\underline{e}_t/\underline{e}_{t-1}) \beta^{-1} \underline{M}^{(\theta-1)} = (\underline{e}_t/\underline{e}_{t-1}) \beta^{-1} \exp((\theta-1)(\underline{\mu} - \frac{1}{2} \gamma \underline{\sigma}^2))$  for the world mutual fund.

<sup>19</sup>Recent papers that examine this type of equilibrium include Baxter and Crucini (1991) and Clarida (1990).

<sup>20</sup>I also calculated the implied parameters from the autarchy returns for each country individually. These parameters varied widely due to substantial differences in equity returns.

returns fall from almost 11% to 7%.

For the closed U.S. economy, the model predicts a value for  $\theta$  of about 2, but a value for  $\gamma$  of almost 53. The difference between these numbers and other estimates in the literature largely derive from differences in the risk-free rate and the equity premium over this period. Using century long data, Mehra and Prescott (1985) report an estimate of 0.9 for the risk-free rate and about 6% for the excess of equity over the risk-free rate. Here, however, the risk-free rate is substantially higher, near 3% and the equity premium is smaller, near 1.6%. This is reflected in a lower  $\theta$  (higher intertemporal substitutability.)

In Panel B, I next examine the parameters necessary to explain the integrated world equilibrium by setting mean equity returns equal to equation (24). This panel reports the means and standard deviations of the world equilibrium when all countries have the utility parameters that match the U.S. equity and risk-free rate. In the integrated equilibrium, mean returns are lower than under autarchy, ranging from 3% for Italy to just above 5% for the UK.

Finally, Panel C gives the parameters necessary to justify the mean equity return on the world mutual fund by setting equation (25) equal to the world equity return of 5.86%. As the estimates show, the risk-aversion parameter is higher at 132.5 while the inverse of the intertemporal elasticity of substitution is about the same at 2.6. The mean implied returns range over almost 4 percentage points from 3.4% for Italy to 7.4% for the U.K.

Notably, even when the means of returns are matched in Table 4, the variances are not. In all cases, the implied standard deviations are significantly lower than the actual standard deviations in the data. I will return to this problem below.

### *(5.2) Implied Welfare Gains from International Diversification*

I now ask whether the gains implied by the equity returns in the partial equilibrium framework are consistent with the gains implied by the general equilibrium framework once utility parameters have been matched to actual equity means. Table 5, Panel A recalculates the welfare gains in Table 2, but instead of arbitrary values of  $\gamma$  and  $\theta$ , uses the parameters in Table 4 that match the equity premium.

The first column gives welfare gain estimates when the utility parameters are set to match



the U.S. returns under autarchy. While the gains for some countries such as Italy, the UK and Canada are quite large, in excess of 33% of permanent consumption, the gain for the U.S. is lower at about 7%. Although this level implies a significantly larger gain for the U.S. than the "plausible" parameters examined in Table 2, it is still lower than the gains of at least 20% suggested by stock returns in Table 1.

The second column uses the parameters matched to the U.S. equity return under open markets. Again, the costs for the U.S. are larger, but remain less than those implied by stock returns at about 12%.

Finally, the third column matches the equity premium to the return on the world equity mutual fund. In this case, with a risk aversion parameter of 132.5, the welfare costs are in the range suggested by stock returns. Furthermore, the risk-adjusted growth rates for the high variance countries of the UK and Canada now become negative and the effective discount rate exceeds unity, implying that utility is not defined.

Overall, the evidence suggests that when utility parameters match the world equity premium, even consumption suggests large gains to risk-sharing.

### *(5.3) Graphical Representation of the Gains with Matched Parameters*

Figure 5 shows why the gains from diversification are so large with high risk aversion. For the purposes of illustration, I use the parameters that match the world equity premium given in the last column to Table 5, Panel A. With high risk aversion, the autarchy consumption profiles are significantly flatter than the low risk aversion counterparts in Figure 3. The unadjusted consumption growth rate for the US is less than the risk-adjustment factor based upon high risk aversion. As a result, risk-adjusted consumption path is downward-sloping.

The risk-sharing equilibrium leads to obvious welfare gains for all countries. With high risk-aversion, investors in each country value greatly small reductions in consumption variances. As a result, the stocks from countries such as Italy having low covariances with the rest of the world become much more valuable. This phenomenon also shows why the US has lower gains than the other countries. Since the US has the highest covariance with the rest of the world, its stock is

relatively cheap. However, the gains from buying an increasing risk-adjusted consumption path significantly outweigh the losses from the low equity value.

Comparing Figures 4 and 5 show that the consumption trade-offs of using stock return means and variances are quite different from those measured using parameters that make consumption means and variances match stock returns. The stock return calculations using low risk aversion imply steeply rising consumption paths. The gains derive from a higher growth rate, an intertemporal substitution toward higher current consumption, or both. On the other hand, high risk aversion lowers the risk-adjusted consumption growth rate. Thus, most of the gains derive from a higher value of domestic equity in world markets than at home.

#### (5.4) Matching Variances

Above, I described the gains from diversification from matching the means of stock returns. However, Table 4 showed that the models imply variances that are too low to be consistent with actual stock return variances. I therefore calculate the utility parameters that match the risk-free rate and the variance of stocks.<sup>21</sup>

Table 5, Panel B reports the results for the three stock returns described previously. As the table shows, the intertemporal substitutability must be very low to match the stock return variances. However, at these levels, the gains from future higher consumption growth become very low and the gains are close to zero.

### Section 6: Conclusion

In this paper, I have examined the sources of differences between consumption-based and stock-return based calculations of the welfare costs of imperfect international risk-sharing. The differences largely come from differences in the utility cost of realized variability. This utility cost can arise in two ways. First, the variability of the consumption stream itself may be high. Thus, when consumption variability is measured from stocks, the high stock variance implies significant variations. Second, the *value* of the reducing the variability may be high. For example, when risk

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<sup>21</sup>The standard deviation of stocks under autarchy is:  $\text{StDev}(R_i^j) = \beta^{-1} \exp(\theta \mu_j - (\theta-1) \frac{1}{2} \gamma \sigma_j^2) \times (\exp(\sigma_j^2) - 1)^{1/4}$ , and under integrated markets is:  $\text{StDev}(\underline{R}_i^j) = \beta^{-1} \exp(\theta \underline{\mu} - (\theta+1) \frac{1}{2} \gamma \underline{\sigma}^2 + \gamma \underline{\sigma}_j) \times (\exp(\underline{\sigma}_j^2) - 1)^{1/4}$ .

aversion is sufficiently high to explain the equity premium on an international diversified portfolio, the implied gains from risk sharing measured from consumption is comparable with the gains based upon stock returns directly.

The paper also shows that some plausible explanations are not important. It has been argued that calculations based upon stock returns that treat these returns as given do not allow the investor to internalize the effects of his actions on the stock return in equilibrium. I show that internalizing these effects can actually increase the gains from diversification, as in the case of Japan.

This paper pinpoints the reasons for significant differences in apparent gains to risk sharing. These results should be useful for understanding the increasingly important question of what the gains actually are.

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### Appendix A: Data Sources and Construction

Stock Market Data: The data for the stock market series were kindly provided by Richard Marston. The series are the country indexes from *Morgan Stanley* with gross dividends reinvested. The series are measured in dollars and converted into December year-over-year growth rates in real U.S. terms by deflating by the consumer price index for all goods from the *Economic Report of the President*. The foreign mutual fund used in Figure 1 and Table 1 is a 1969 population-weighted average of the non-US G-7 countries excluding Italy. The stock on Italy was excluded since, with its negative return and higher variance than the U.S. stock return, made it clearly dominated by the other stocks. The risk-free rate is the six month London Interbank Offer Rate (LIBOR) from the *London Financial Times* supplied by WEFA.

Consumption Data: Following standard practice in the literature, the consumption data were taken from the Penn World Tables described in Summers and Heston (1991). Similarly, the endowment series was also taken to be consumption to correspond to the assumption of no capital investment. These series were updated using the most recent data available in Mark 5.6 of the data set.

### Appendix B: Monte Carlo Evidence of Log-Normality in World Endowment Process

In the text, I assume the world *per capita* endowment process can be approximated by a log normal distribution while the individual country endowments also follow log normal distributions. This appendix describes Monte Carlo evidence to study the plausibility of this assumption.

To generate the implied world endowment process,  $\underline{e}_t$ , I first generate sequences of individual country endowments,  $e_t^j$ . To parameterize the distribution, I estimate the Cholesky decomposition of the variance-covariance matrix of the vector of:  $\{e_t^1, e_t^2, \dots, e_t^7\}' = \mathbf{e}_t$ . Thus, the Cholesky factor is  $H$ , where  $E(\mathbf{e}_t \mathbf{e}_t') = H H'$ . Then, using  $H$  and  $\mu$ , the vector of mean growth rates, the following steps were taken. (i) Generate a seven dimensional standard normal variable. Construct pseudo country endowments:  $\mathbf{e}_t$ . (ii) Update to generate number of years by number of countries (22 x 7) matrix of endowments. (iii) For each time  $t$ , calculate the world *per capita* endowment by forming:  $\underline{e}_t = \sum_{j=1}^7 \pi_j^t e_t^j$  where  $\pi_j^t$  is the share of country  $j$  in world population. (iv) Repeat 4000 times, generating  $4000 \times 22 = 88,000$  numbers of observations. (v) Take the natural

logarithm of these variables and form growth rates.

With this distribution of world endowment growth rates, I calculated some simple diagnostics. First, a histogram confirmed that the distribution looked fairly normal. Second, I calculated a measure of skewness:  $E((\underline{e}_t - \text{mean}(\underline{e}_t))^3)/\sigma^3 = -0.0003$  with a standard deviation of 0.4169. With no skewness, this number should equal zero. Therefore, the distribution is insignificantly different from a distribution with no skewness. Third, I calculate a measure of kurtosis:  $E((\underline{e}_t - \text{mean}(\underline{e}_t))^4)/\sigma^4 = 2.52$  with a standard deviation of 0.68. The t-statistic of the hypothesis that the kurtosis equals 3 as it should under the normal distribution is -0.7081. Thus, the distribution is insignificantly different from a normal distribution. In sum, all of these three measures suggest the approximation is plausible.



Figure 1: Risk Return Tradeoff for US Investor

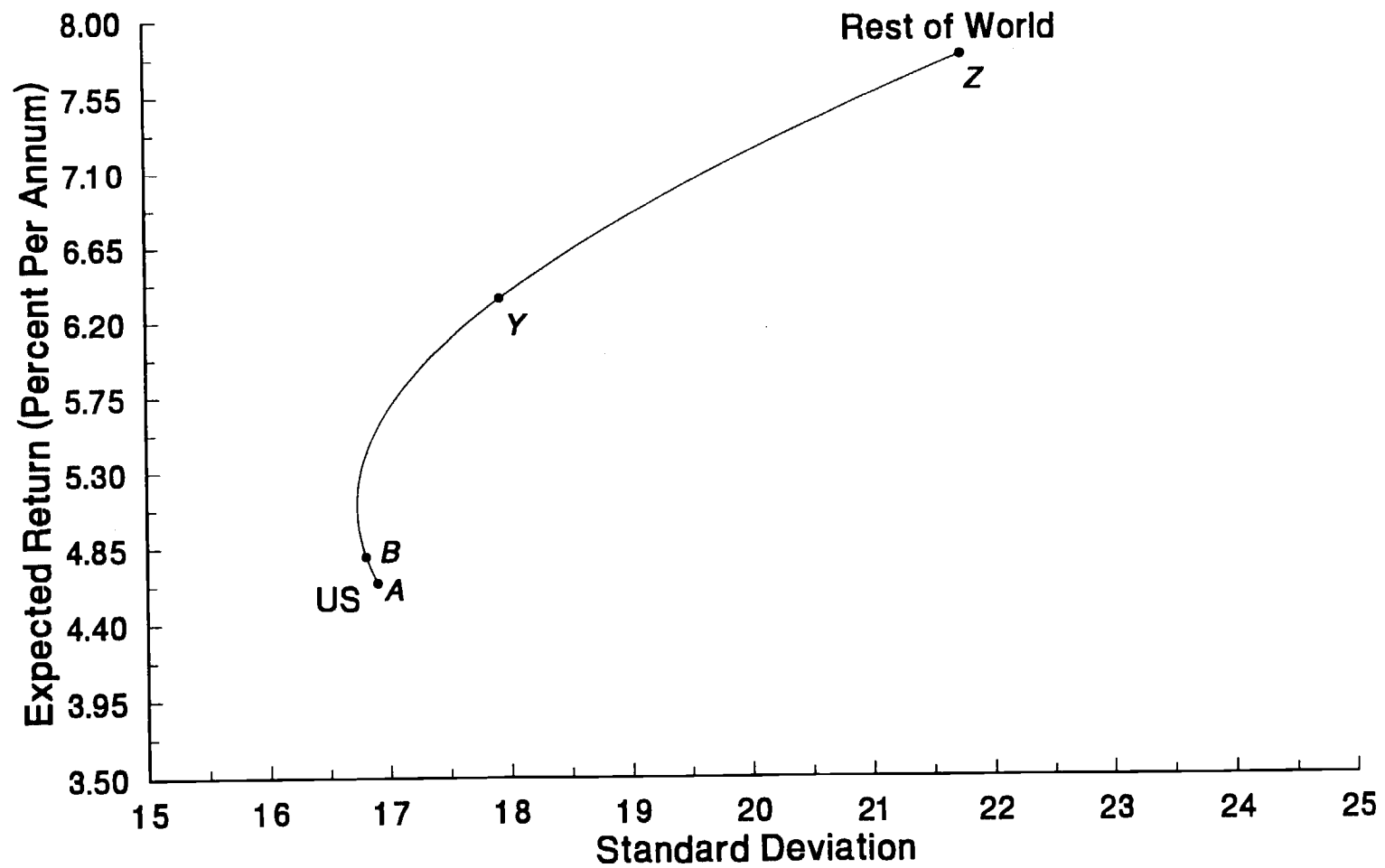
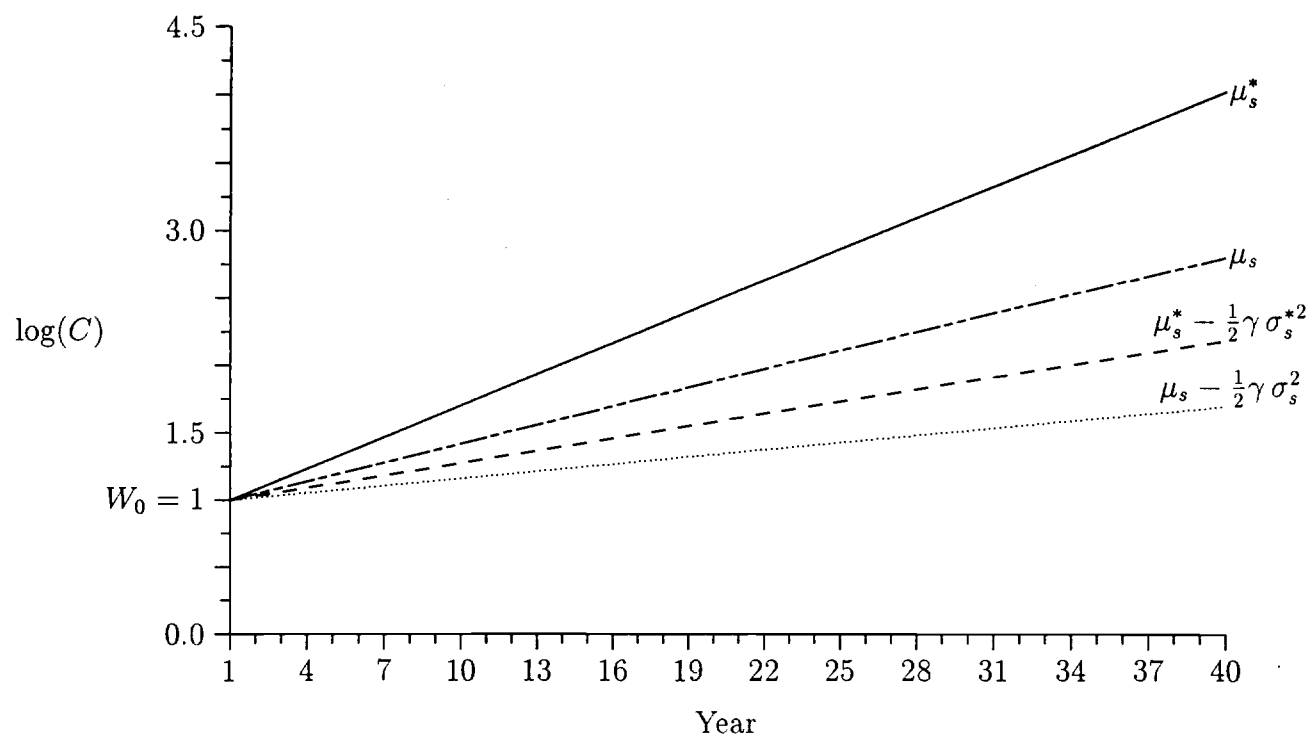
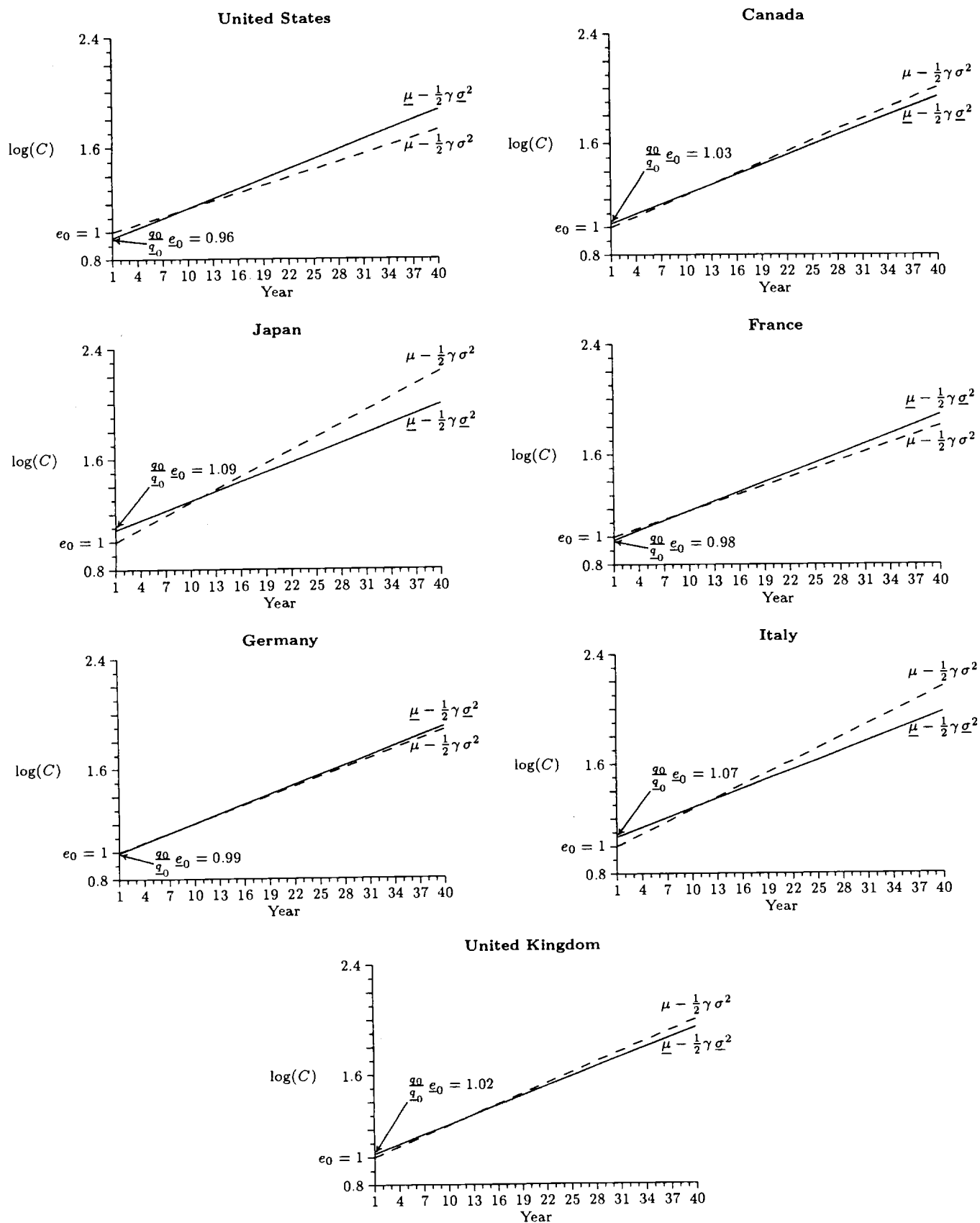


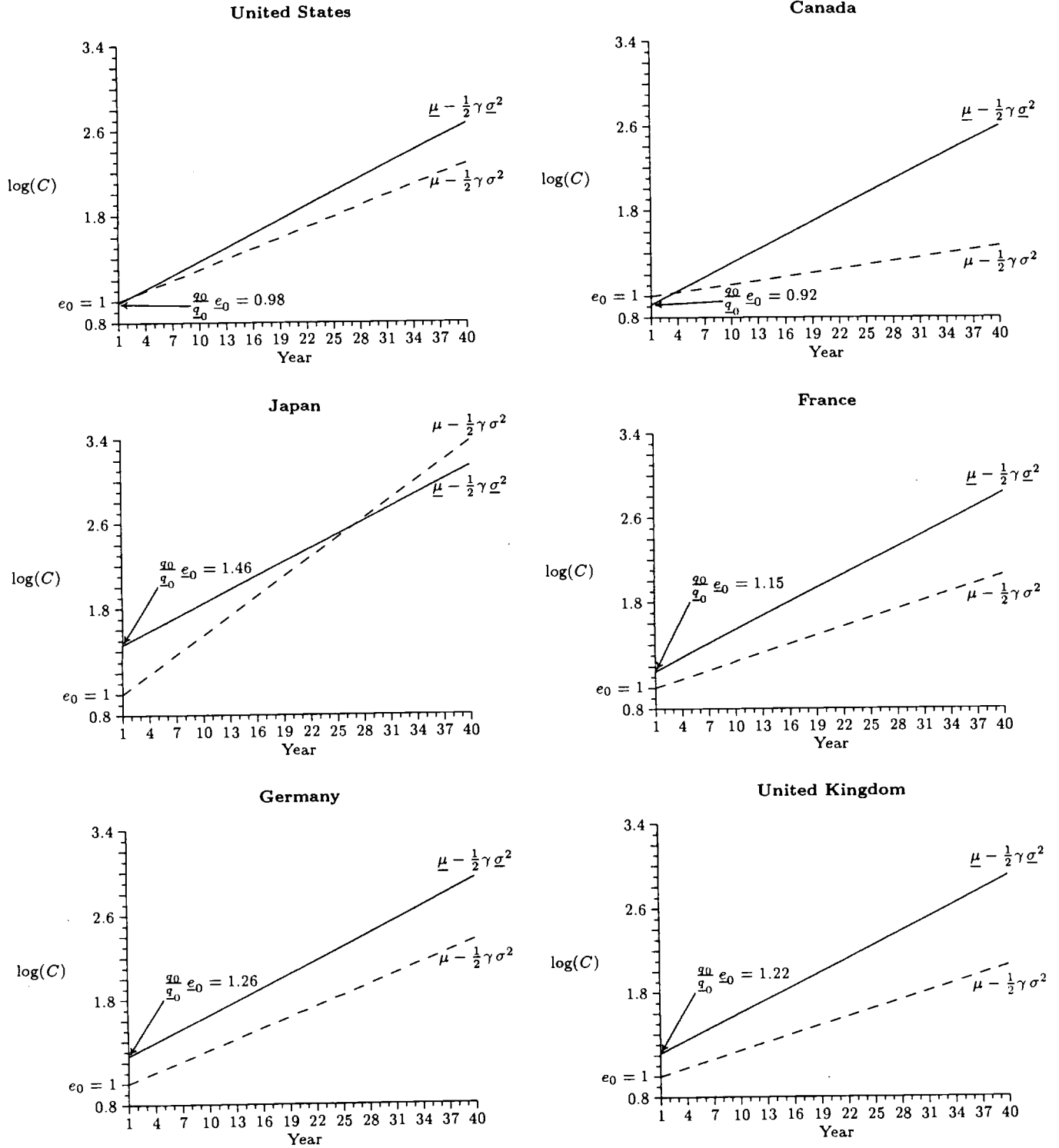
Figure 2: Log Consumption Profile for U.S. Investor  
Facing Exogenous Stock Returns



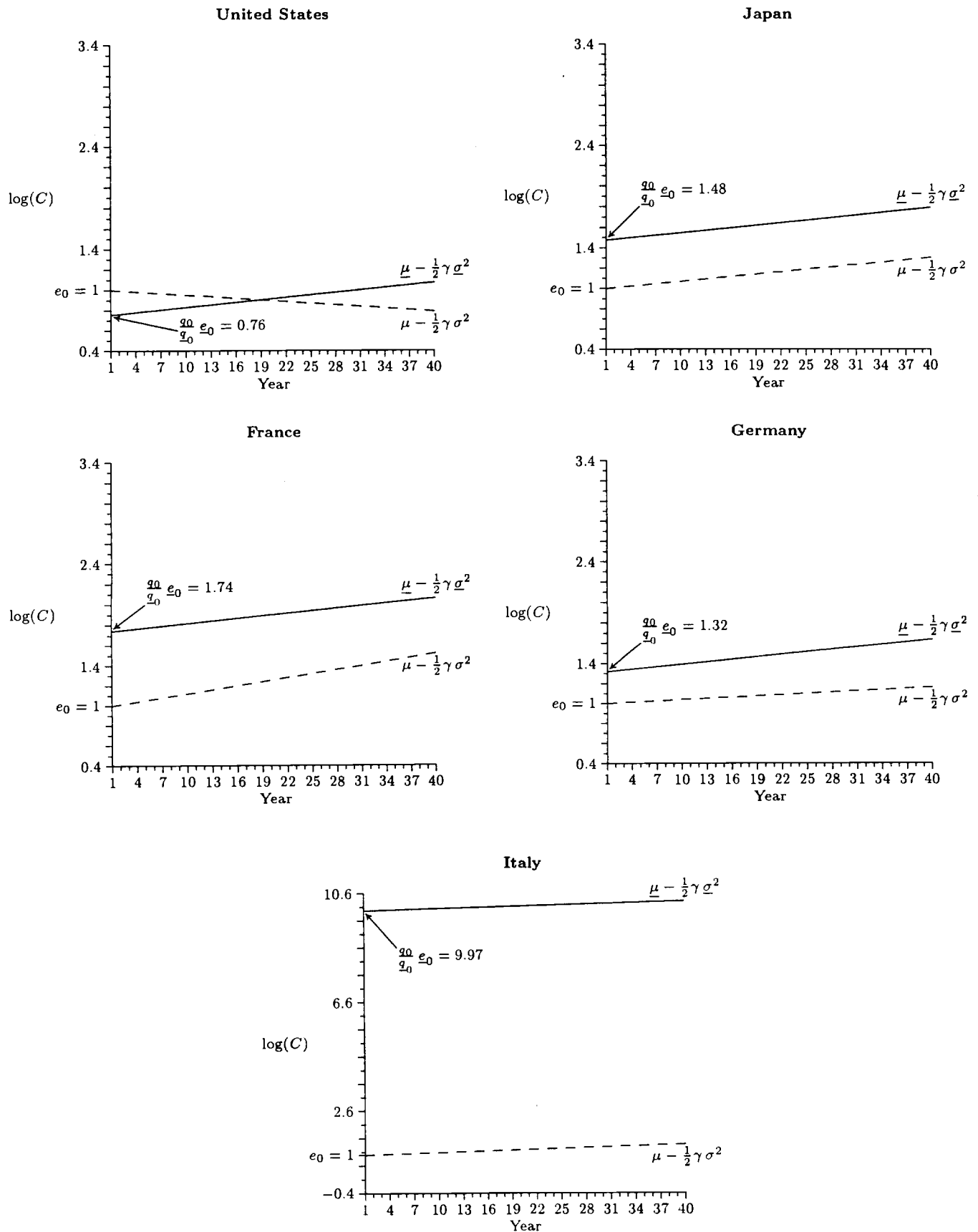
**Figure 3: Risk-Adjusted Consumption Profile for Endogenous Stock Returns**  
 Using Consumption Means and Variances ( $\gamma = 2, \theta = 5$ )



**Figure 4: Risk-Adjusted Consumption Profile for Endogenous Stock Returns**  
Using Stock Return Means and Variances ( $\gamma = 2, \theta = 5$ )



**Figure 5: Risk-Adjusted Consumption Profile for Endogenous Stock Returns  
Using Consumption Means and Variances ( $\gamma = 132.5, \theta = 2.6$ )**



**Table 1**  
**Partial Equilibrium Gains From US Diversification**  
**Using Stock Returns**

A. Time Additive Utility				
	$\theta = 2$	$\theta = 3$	$\theta = 4$	$\theta = 5$
$\gamma = \theta$	28.83	18.20	21.75	NA
B. Epstein-Zin-Weil Utility				
	$\theta = 2$	$\theta = 3$	$\theta = 4$	$\theta = 5$
$\gamma = 2$	28.83	18.03	12.96	10.02
$\gamma = 3$	26.97	18.20	13.65	10.87
$\gamma = 4$	32.39	26.00	21.75	18.70
$\gamma = 5$	51.70	NA	NA	NA
C. Summary Statistics				
	<u>Mean</u>	<u>Standard Deviation</u>	<u>Correlation</u>	
US	4.64	16.90	0.673	
Foreign	7.78	21.75	0.673	

**Notes:** "NA" means not available since discount rate is negative.

**Table 2**  
**General Equilibrium Gains From Diversification**  
**Using Consumption**

A. Summary Statistics										
	Mean	Standard Deviation	Correlation Matrix							
			Canada	US	Japan	France	Germany	Italy	UK	World
Canada	2.61	2.70	1.00	0.62	0.20	0.24	0.42	0.32	0.34	0.62
US	1.86	1.91	—	1.00	0.57	0.50	0.54	0.02	0.63	0.95
Japan	3.18	1.91	—	—	1.00	0.73	0.45	0.11	0.68	0.74
France	2.07	1.06	—	—	—	1.00	0.58	0.17	0.46	0.65
Germany	2.26	1.67	—	—	—	—	1.00	0.26	0.39	0.65
Italy	2.98	1.76	—	—	—	—	—	1.00	0.18	0.21
UK	2.58	2.93	—	—	—	—	—	—	1.00	0.77
World	2.34	1.53	—	—	—	—	—	—	—	1.00

B. Same Means					
	$\theta = 5$	$\theta = 5$	$\theta = 2$	$\theta = 2$	
	$\gamma = 2$	$\gamma = 5$	$\gamma = 2$	$\gamma = 5$	
US	0.04	0.08	0.10	0.25	
Canada	0.39	0.97	1.01	2.47	
Japan	0.15	0.36	0.38	0.95	
France	0.13	0.34	0.32	0.83	
Germany	0.17	0.42	0.42	1.05	
Italy	0.40	1.00	1.01	2.53	
UK	0.35	0.86	0.92	2.30	

C. Differing Means					
	$\theta = 5$	$\theta = 5$	$\theta = 2$	$\theta = 2$	
	$\gamma = 2$	$\gamma = 5$	$\gamma = 2$	$\gamma = 5$	
US	0.06	0.16	0.85	1.11	
Canada	0.79	1.30	1.68	3.09	
Japan	2.16	2.36	4.91	5.47	
France	0.00	0.19	0.40	0.83	
Germany	0.10	0.35	0.37	1.00	
Italy	1.76	2.38	3.91	5.60	
UK	0.68	1.11	1.44	2.66	

**Table 3**

**Estimates of  $\gamma$  and  $\theta$  from World Portfolio**

A. Summary Statistics									
	US	Canada	Japan	France	Germany	Italy	UK	World	Interest Rate
Mean	4.64	2.64	10.72	6.48	6.34	-0.17	7.02	5.86	2.98
Standard Deviation	16.51	16.94	30.46	27.45	23.88	32.96	29.36	17.61	2.62
Corr( $X$ , World)	0.873	0.735	0.791	0.689	0.510	0.615	0.672	1.000	—
B. Differing Stock $\mu$ , Stock $\Sigma$									
	$\theta = 5$			$\theta = 2$					
	$\gamma = 2$			$\gamma = 5$					
US	3.85			11.99					
Canada	14.80			35.38					
Japan	26.49			87.08					
France	21.10			53.50					
Germany	24.04			64.33					
Italy	NA			NA					
UK	25.18			63.12					



**Table 4**

**Estimates of  $\gamma$  and  $\theta$  from World Portfolio**

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A. Implied Returns Matching US Equity for Autarchy								
<i>Implied Parameters: <math>\theta = 1.69</math>, <math>\gamma = 52.72</math></i>								
<i>Implied Returns:</i>								
	<u>US</u>	<u>Canada</u>	<u>Japan</u>	<u>France</u>	<u>Germany</u>	<u>Italy</u>	<u>UK</u>	<u>World</u>
Mean	4.64	5.31	7.00	5.47	5.51	6.75	5.02	5.01
Standard Deviation	2.00	2.82	2.04	1.12	1.77	1.87	3.08	1.60

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B. Matching US Equity for Integrated Markets and Risk Free Rate								
<i>Implied Parameters: <math>\theta = 0.91</math>, <math>\gamma = 69.8</math></i>								
<i>Implied Returns:</i>								
	<u>US</u>	<u>Canada</u>	<u>Japan</u>	<u>France</u>	<u>Germany</u>	<u>Italy</u>	<u>UK</u>	<u>World</u>
Mean	4.64	4.50	4.20	3.40	3.84	3.04	5.13	4.32
Standard Deviation	2.00	2.82	1.99	1.10	1.74	1.81	3.08	1.59

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C. Matching World Equity and Risk Free Rate								
<i>Implied Parameters: <math>\theta = 2.6</math>, <math>\gamma = 132.5</math></i>								
<i>Implied Returns:</i>								
	<u>US</u>	<u>Canada</u>	<u>Japan</u>	<u>France</u>	<u>Germany</u>	<u>Italy</u>	<u>UK</u>	<u>World</u>
Mean	6.47	6.20	5.63	4.09	4.93	3.40	7.42	5.86
Standard Deviation	2.04	2.87	2.02	1.10	1.76	1.82	3.15	1.61
Corr ( $X$ , World)	0.623	0.947	0.742	0.651	0.652	0.206	0.767	1.000

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**Table 5**

**General Equilibrium Gain From Diversification  
Using Implied Parameters**

	A. Matching Means of:		
	US Equity -Autarchy	US Equity -Integrated	World Equity
	$\gamma = 52.72$	$\gamma = 69.8$	$\gamma = 132.5$
	$\theta = 1.69$	$\theta = 0.91$	$\theta = 2.62$
US	7.3	12.4	31.4
Canada	33.2	55.2	NA
Japan	18.3	40.3	32.8
France	11.3	27.5	34.3
Germany	14.2	30.1	33.4
Italy	42.4	NA	89.5
UK	33.5	51.7	NA
	B. Matching Variances of:		
	US Equity -Autarchy	US Equity -Integrated	World Equity
	$\gamma = 5.94$	$\gamma = 8.39$	$\gamma = 11.98$
	$\theta = 123.96$	$\theta = 94.61$	$\theta = 109.55$
US	0.00	0.00	0.00
Canada	0.29	0.30	0.30
Japan	0.90	0.90	0.87
France	0.00	0.00	0.00
Germany	0.00	0.00	0.00
Italy	0.77	0.84	0.90
UK	0.22	0.20	0.17